Quark model study of the semileptonic $B \rightarrow \pi$ decay

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Abstract. The semileptonic decay $B \rightarrow \pi l \bar{\nu}_l$ is studied starting from a simple quark model and taking into account the effect of the B^* resonance. A novel, multiply subtracted, Omnès dispersion relation has been implemented to extend the predictions of the quark model to all physical q^2 values. We find $|V_{ub}| = 0.0034 \pm 0.0003(\mathrm{exp.}) \pm 0.0007(\mathrm{theory})$, in good agreement with experiment.

Keywords: Decay of bottom mesons, nonrelativistic quark model, dispersion relations, Kobayashi-Maskawa matrix elements
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INTRODUCTION

The measurement of the exclusive semileptonic decay $B \rightarrow \pi l \bar{\nu}_l$ can be used to determine de Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$. With no flavor symmetry constraining the hadronic matrix elements, the errors on $|V_{ub}|$ are currently dominated by theoretical uncertainties, being a determination of $|V_{ub}|$, with well understood uncertainties, a priority of heavy flavor physics. The application of Watson's theorem to the $B \rightarrow \pi$ semileptonic decay allows one to write a dispersion relation for each of the form factors entering in the hadronic matrix element. This leads to the so-called Omnès representation, which can be used to constrain the q^2 dependence of the form factors assuming some knowledge of the elastic $\pi B \rightarrow \pi B$ scattering amplitudes. The use of multiple subtractions will allow to combine predictions from various methods in different q^2 regions. In this talk, we show how this Omnès scheme can be used to combine the results at $q^2 = 0$ of the relevant hadron $B \rightarrow \pi l \bar{\nu}_l$ form factors from light cone sum rules (LCSR) calculations, with those obtained from a simple nonrelativistic constituent quark model (NRCQM) in its region of applicability, near zero recoil. In this way we end up, with an accurate description of the differential decay rate, except for V_{ub}, in the whole physically accessible q^2 range. We also use a Monte Carlo simulation to estimate the theoretical error bands of our procedure.

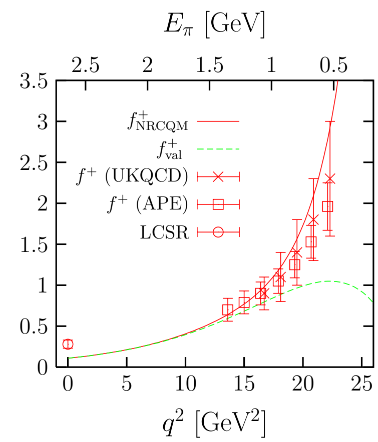

FIGURE 1. Valence quark (val), and valence quark plus B^* contribution (NRCQM) to f^+. We also plot lattice QCD and LCSR f^+ results. See Ref. [1] for details.

NRCQM: VALENCE QUARK, B^* RESONANCE AND OMNÈS REPRESENTATION

The hadronic matrix element for $B^0 \to \pi^- l^+ \nu_l$ can be parametrized in terms of two dimensionless form factors, f^+ and f^0, of which only f^+ contributes for massless final leptons. In Figure 1 we show under "*val*" the NRCQM prediction for f^+ when considering only the valence quark contribution. The description fails in the whole q^2 range: from the region close to $q^2_{\rm max}$, where a nonrelativistic model should work best, to the opposite end where the pion is ultrarelativistic and thus predictions from a nonrelativistic scheme are unreliable. As first pointed out in Ref. [2], near zero recoil the $B \to \pi l^+ \nu_l$ decay is dominated by the effects of the B^* resonance, which is quite close to $q^2_{\rm max}$. These effects of the B^* resonance must be added as a distinct coherent contribution. We have consistently evaluated within our model the B^* resonance contribution to the f^+ form factor (See Ref. [1] and references therein for details). The result is that the B^* resonance plays a role only near q^2_{max}, being strongly suppressed by a soft hadronic vertex outside that region. The inclusion of the B^* resonance contribution to the form factor (f^+_{NRCQM} in the figure) improves the simple valence quark contribution down to values around 15 GeV^2. Below that the description is still poor.

We have now used the Omnès representation to combine the NRCQM predictions at high q^2 with the LCSR result at $q^2 = 0$. This representation requires as an input the elastic $\pi B \to \pi B$ phase shift $\delta(s)$ in the $J^P = 1^-$ and isospin $I = 1/2$ channel plus the form factor at different q^2 values below the πB threshold where we will perform the subtractions. With a large enough number of subtractions only the phase shift at or near threshold is needed. We can then approximate $\delta(s) \approx \pi$ (Levinson's theorem) which renders our calculation analytic.

In Figure 2 we show with a solid line the form factor obtained using the Omnès representation. We have used as subtraction points five q^2 values between 18 GeV^2 and

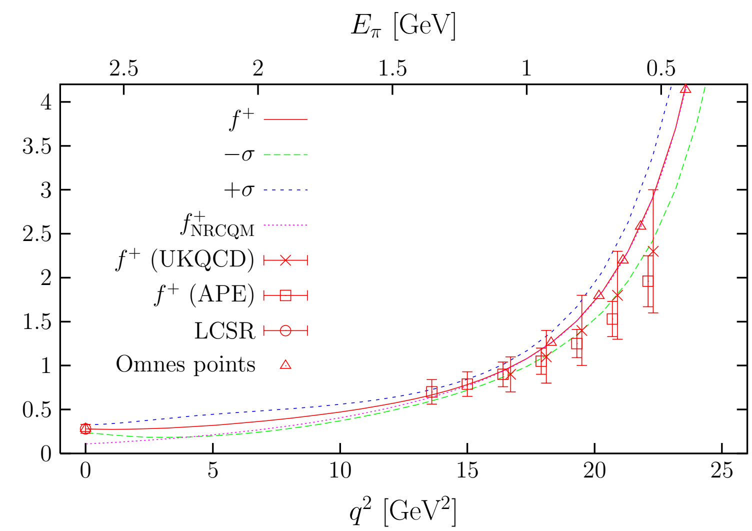

FIGURE 2. The solid line represents the Omnès improved form factor. The subtraction points are denoted by triangles. The $\pm\sigma$ lines show the theoretical uncertainty band on the Omnès form factor. We compare with previous lattice results.

q^2_{max}, for which the NRCQM predictions (valence+pole) have been used, plus the LCSR prediction at $q^2 = 0$. We have paid special attention to the estimation of theoretical uncertainties that come from two main sources: (i) uncertainties in the quark–antiquark nonrelativistic interaction and (ii) uncertainties on the $[g_{B^*B\pi} f_{B^*}]$ product and on the input to the multiply subtracted Omnès representation. As a result, we obtain the 68% confidence level region enclosed between $\pm\sigma$ lines. See Ref. [1] for details. Comparing with the experimental decay width, we obtain $|V_{ub}| = 0.0034 \pm 0.0003(\mathrm{exp.}) \pm 0.0007(\mathrm{theo.})$ in good agreement with a recent experimental determination by the CLEO Collaboration [3].

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